Poisson

Parametrisation

The Poisson distribution is

$$\text{Prob}(y) = \frac{\lambda^y}{y!} \exp(-\lambda)$$

for responses $y = 0, 1, 2, \ldots$, where

λ: the expected value.

Link-function

The mean and variance of y are given as

$$\mu = \lambda \qquad \text{and} \qquad \sigma^2 = \lambda$$

and the mean is linked to the linear predictor by

$$\lambda(\eta) = E \exp(\eta)$$

where $E > 0$ is a known constant (or $\log(E)$ is the offset of η).

Hyperparameters

None.

Specification

- family = `poisson`
- Required arguments: (integer-valued) y and E

There is an alternative expert-version,

- family = `xpoisson`
- Required arguments: y and E

which allows the Poisson likelihood to be evaluated for a real-numbered response $y \geq 0$, in cases where this is known to make sense. Note that $y!$ is computed using the integer part of y.

Example

In the following example we estimate the parameters in a simulated example with Poisson responses.

```
n=100
a = 1
b = 1
z = rnorm(n)
eta = a + b*z
E = sample(1:10, n, replace=TRUE)
lambda = E*exp(eta)
y = rpois(n, lambda = lambda)

data = list(y=y,z=z)
formula = y ~ 1+z
result = inla(formula, family = "poisson", data = data, E=E)
summary(result)
```

Notes

This likelihood also accept $E = 0$ and in this case $\log(E)$ is *defined* to be 0. Non-integer values of $y \geq 0$ is accepted although the normalising constant of the likelihood is then wrong (but its a constant only).

For the quantile-link, then `model="quantile"` is applied to λ only and this is then scaled with `E`. A more population version, can be achived moving the constant `E` into the linear predictor by

```
~ offset(log(E)) + ...
```

Note there is no link-model `pquantile` for the Poisson, as this would disable the `E` argument.